Yau College Math Competition 2023

Final Probability and Statistics

Individual All-round Problems (June 10-11, 2023)

Choose 1 from the following 2 problems.

Problem 1. Suppose that $X_1, X_2, \cdots$ are i.i.d. random variables such that $\mathrm{P}(X_i = 1) = \mathrm{P}(X_i = -1) = \frac{1}{2}$. Take some $a \in (0, 1)$.

(1) Prove or disprove that the distribution function of $\xi = \sum_{k=1}^{\infty} a^k X_k$ is continuous.

(2) For $a = \frac{1}{2}$, find the distribution of ξ.

(3) For $a = \frac{1}{3}$, find the distribution of ξ.

Problem 2. Let $F_2 = \{0, 1\}$ be the number field with two elements and F_2^d be the d-dimensional vector space over F_2. Let $t \leq d$ and $X_k, k = 1, 2, \cdots, t$, be i.i.d random variables uniformly distributed on F_2^d.

(1) Find the probability that $X_1, X_2, \cdots, X_t$ are linearly independent (as vectors in F_2^d).

(2) Give a positive lower bound (independent of t, and as large as you can) of the above probability.